`idescent.prob`	*Probability of identical-by-descent*

Usage

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idescent.prob(recomb.prob, like.idescent, like.no.idescent)
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Arguments

`recomb.prob` vector of probabilities for recombination out of identical-by-descent from preceding identical-by-descent position

`like.idecent` probability of observed evidence at positions assuming identical-by-descent at positions

`like.no.idecent`
probability of observed evidence at positions assuming NOT identical-by-descent at positions

Math

recomb.prob:
Define I_k to be the event that nucleotide at position k is identical-by-decent (IBD).
The argument **recomb.prob** corresponds to vector ρ with definition

$$\rho_k = \Pr(\overline{I_k}|I_{k-1}) = \Pr(\overline{I_{k-1}}|I_k)$$

You can use the function **recombination.prob** R function to calculate such recombination probabilities.
For convenience, position zero is modeled as a position infinitely far away such that for all k

$$\rho_1 = \Pr(\overline{I_1}|I_0) = \Pr(\overline{I_k})$$

$$\Pr(I_k) = 1 - \rho_1$$

This is the unconditional probability that a nucleotide is not identical-by-descent.
A useful formula is

$$\Pr(I_k|\overline{I_{k-1}}) = \rho_k(1 - \rho_1)/\rho_1$$

like.idescent and like.no.idescent:
Let W_k be the event of evidence observed for nucleotides at position k.
The argument **like.idescent** corresponds to vector ϕ with definition

$$\phi_k = \Pr(W_k|I_k)$$

and argument **like.no.idescent** corresponds to vector ψ with definition

$$\psi_k = \Pr(W_k|\overline{I_k})$$

The function **haploids.likeli.idescent** can be used to generate **like.idescent** values. For **like.no.idescent**, the default of just **0.5** is an OK default for comparing one sequence (halploid) to one other sequence (haploid).

one-sided conditional identical-by-descent probability:
We define the event of evidence observed for nucleotides **after** position k:

$$A_k = \bigcap_{i>k} W_i$$

Likewise, we define the event of evidence observed for nucleotides **before** position k:

$$B_k = \bigcap_{i<k} W_i$$

Note that B_1 is trivially the event of all possibilities (i.e. there is no W_0).
The output of R function **roll.idescent.prob** is vector $\Pr(I_i|B_i)$.
We assume that events W_i and B_i are conditionally independent given $J \in \{I_i, \overline{I_i}\}$, that is

$$\Pr(W_i \cap B_i|J) = \Pr(W_i|J)\Pr(B_i|J)$$

which implies

$$\Pr(W_i|J) = \frac{\Pr(W_i \cap B_i|J)}{\Pr(B_i|J)} = \Pr(W_i|B_i \cap J)$$

$$\Pr(W_i|J)\Pr(J|B_i) = \Pr(W_i|B_i \cap J)\Pr(J|B_i) = \Pr(W_i \cap J|B_i)$$

which can be applied to calculate

$$\Pr(W_i \cap I_i|B_i) + \Pr(W_i \cap \overline{I_i}|B_i) = \phi_i \Pr(I_i|B_i) + \psi_i \Pr(\overline{I_i}|B_i)$$
$$\Pr(W_i|B_i) = \phi_i \Pr(I_i|B_i) + \psi_i(1 - \Pr(I_i|B_i))$$
$$\Pr(I_i|W_i \cap B_i) = \frac{\Pr(W_i \cap I_i|B_i)}{\Pr(W_i|B_i)}$$
$$\Pr(I_i|B_{i+1}) = \frac{\phi_i \Pr(I_i|B_i)}{\phi_i \Pr(I_i|B_i) + \psi_i(1 - \Pr(I_i|B_i))}$$

We assume B_{i+1} is independent of I_{i+1} for any given $J \in \{I_i, \overline{I_i}\}$, that is

$$\Pr(I_{i+1} \cap B_{i+1}|J) = \Pr(I_{i+1}|J)\Pr(B_{i+1}|J)$$

thus

$$\Pr(I_{i+1}|J \cap B_{i+1}) = \Pr(I_{i+1}|J)$$

Applying this to the expression of $\Pr(I_{i+1}|B_{i+1})$ we get

$$\Pr(I_{i+1}|B_{i+1}) = \Pr(I_{i+1} \cap I_i|B_{i+1}) + \Pr(I_{i+1} \cap \overline{I_i}|B_{i+1})$$

$$\Pr(I_{i+1}|B_{i+1}) = \Pr(I_{i+1}|I_i \cap B_{i+1})\Pr(I_i|B_{i+1}) + \Pr(I_{i+1}|\overline{I_i} \cap B_{i+1})\Pr(\overline{I_i}|B_{i+1})$$

$$\Pr(I_{i+1}|B_{i+1}) = \Pr(I_{i+1}|I_i)\Pr(I_i|B_{i+1}) + \Pr(I_{i+1}|\overline{I_i})\Pr(\overline{I_i}|B_{i+1})$$

$$\Pr(I_{i+1}|B_{i+1}) = (1 - \rho_{i+1})\Pr(I_i|B_{i+1}) + (\rho_{i+1}(1 - \rho_1)/\rho_1)(1 - \Pr(I_i|B_{i+1}))$$

idenscent.prob:

We assume that $A = A_k$ and $B = B_k$ are conditionally independent given fixed $I = I_k$. That is, if the nucleotide at position k is inherited identically, then the observed nucleotides before and after are independent of each other.

$$\Pr(A \cap B|I) = \Pr(A|I)\Pr(B|I)$$

therefore

$$\frac{\Pr(A \cap B|I)\Pr(I)}{\Pr(A)\Pr(B)} = \frac{\Pr(A|I)\Pr(B|I)\Pr(I)^2}{\Pr(A)\Pr(B)\Pr(I)}$$

$$\frac{\Pr(A \cap B \cap I)}{\Pr(A)\Pr(B)} = \frac{\Pr(I|A)\Pr(I|B)}{\Pr(I)}$$

$$\frac{\Pr(A \cap B)}{\Pr(A)\Pr(B)} = \frac{\Pr(I|A)\Pr(I|B)}{\Pr(I)} + \frac{\Pr(\bar{I}|A)\Pr(\bar{I}|B)}{\Pr(\bar{I})}$$

$$\Pr(I|A \cap B) = \frac{\Pr(I|A)\Pr(I|B)/\Pr(I)}{\Pr(I|A)\Pr(I|B)/\Pr(I) + \Pr(\bar{I}|A)\Pr(\bar{I}|B)/\Pr(\bar{I})}$$

We assume $W = W_k$ and $A \cap B$ are conditionally independent given $J \in \{I_k, \overline{I_k}\}$, that is

$$\Pr(W \cap A \cap B|J) = \Pr(W|J)\Pr(A \cap B|J)$$

and thus

$$\Pr(W|J) = \frac{\Pr(W \cap A \cap B|J)}{\Pr(A \cap B|J)} = \Pr(W|J \cap A \cap B)$$

$$\Pr(J|A \cap B)\Pr(W|J) = \Pr(J|A \cap B)\Pr(W|J \cap A \cap B) = \Pr(J \cap W|A \cap B)$$

Finally,

$$\Pr(I \cap W|A \cap B) + \Pr(\bar{I} \cap W|A \cap B) = \Pr(I|A \cap B)\phi_i + \Pr(\bar{I}|A \cap B)\psi_i$$

with which we can get

$$\Pr(I|W \cap A \cap B) = \frac{\Pr(I \cap W|A \cap B)}{\Pr(W|A \cap B)} = \frac{\Pr(I|A \cap B)\phi_i}{\Pr(I|A \cap B)\phi_i + \Pr(\bar{I}|A \cap B)\psi_i}$$

which is the probability of identity-by-descent given all available observed evidence.